BC Calculus. §3-7 Implicit

Pg 162 #1, 5, 7, 14, 23, 24, 31, 35, 37, 44, 52, 56

(1) $x^2y + xy^2 = 6$

$2xy + x^2\frac{dy}{dx} + y^2 + 2xy\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-(2xy + y^2)}{x^2 + 2xy}$

(5) $x = \tan y$

$1 = \sec^2 y \frac{dy}{dx}$

$\frac{dy}{dx} = \cos^2 y$

(7) $x + \tan(xy) = 0$

$1 + \sec^2(xy)\left(y + x\frac{dy}{dx}\right) = 0$

$1 + y\cdot\sec^2(xy) + x\cdot\sec^2(xy)\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-(1 + y\cdot\sec^2(xy))}{x\cdot\sec^2(xy)}$

(14) $x^2 - y^2 = 9, \ (-1, 3)$

$2x - y^2 + 2x^2 y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{y}{x}$

$\left.\frac{dy}{dx}\right|_{(-1,3)} = 3$

a) $y - 3 = 3(x+1)$

b) $y - 3 = -\frac{1}{3}(x+1)$

(23) $2xy + \pi\sin y = 2\pi \quad (1, \frac{\pi}{2})$

$2y + 2x\frac{dy}{dx} + \pi\cos y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2y}{2x + \pi\cos y}$

$\left.\frac{dy}{dx}\right|_{(1,\frac{\pi}{2})} = \frac{-\pi}{2 + 0} = -\frac{\pi}{2}$

a) $y - \frac{\pi}{2} = -\frac{\pi}{2}(x-1)$

b) $y - \frac{\pi}{2} = \frac{2}{\pi}(x-1)$

(24) $y^2 = x^2 + 2x$

$2y\frac{dy}{dx} = 2x + 2$

$\frac{dy}{dx} = \frac{x+1}{y}$

$\frac{d^2y}{dx^2} = \frac{y - \frac{dy}{dx}(x+1)}{y^2}$

$= \frac{y - \left(\frac{x+1}{y}\right)(x+1)}{y^2}$

$= \frac{y^2 - (x+1)^2}{y^3}$

(31) $y = x^{9/4}$

$$\frac{dy}{dx} = \frac{9}{4} x^{5/4}$$

(35) $y = (2x+5)^{-1/2}$

$$\frac{dy}{dx} = -\frac{1}{2}(2x+5)^{-3/2}(2)$$

$$= -(2x+5)^{-3/2}$$

(37) $y = x \cdot (x^2+1)^{1/2}$

$$\frac{dy}{dx} = \sqrt{x^2+1} + \frac{1}{2} x \cdot (x^2+1)^{-1/2}(2x)$$

$$= \sqrt{x^2+1} + \frac{x^2}{\sqrt{x^2+1}}$$

$$= \frac{2x^2+1}{\sqrt{x^2+1}}$$

(49) $x^2 + xy + y^2 = 7$

crosses x-axis when $y = 0$

so $x = \pm\sqrt{7}$

$$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{(2x+y)}{x+2y}$$

$$\left.\frac{dy}{dx}\right|_{(\sqrt{7},0)} = -\frac{2\sqrt{7}}{\sqrt{7}} = -2$$

$$\left.\frac{dy}{dx}\right|_{(-\sqrt{7},0)} = \frac{2\sqrt{7}}{-\sqrt{7}} = -2$$

} same!

(52) $s = (4+6t)^{3/2}$

$$v(t) = \frac{3}{2}(4+6t)^{1/2} \cdot 6$$

$$= 9(4+6t)^{1/2}$$

$$v(2) = 36 \text{ m/s}$$

$$a(t) = \frac{9}{2}(4+6t)^{-1/2} \cdot 6$$

$$= 27(4+6t)^{-1/2}$$

$$a(2) = \frac{27}{4} \text{ m/s}^2$$

(56) $x^2 + 2xy - y - 3y^2 = 0$

$2x + 2y + 2x\frac{dy}{dx} - 6y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-(x+y)}{x-3y}$

$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{-2}{-2} = 1$

slope normal $= -1$

equation of normal:

$y - 1 = -(x-1)$

$y = -x + 2$

Put $y = x + 2$ back in original to get...

$x^2 + 2x(-x+2) - 3(-x+2)^2 = 0$

$x^2 - 2x^2 + 4x - 3x^2 + 12x - 12 = 0$

$-4x^2 + 16x - 12 = 0$

$-4(x^2 + 4x - 3) = 0$

$-4(x-1)(x-3) = 0$

$x = 1, \quad x = 3$

If $x = 1$, $y = -1 + 2 = 1$

so $(1,1)$ is common, but we already knew that

If $x = 3$, $y = -3 + 2 = -1$

so $(3,-1)$ is the other point!